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AN INVENTORY MODEL FOR AN ITEM WITH WEIBULL DETERIORATION AND EXPONENTIALLY REDUCE DEMAND

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Abstract:

This review proposes an EOQ inventory numerical model for deteriorating items with dramatically diminishing interest. In the model, the deficiencies are permitted and to some extent put in a rain check for. The accumulating rate is variable and ward on the hanging tight an ideal opportunity for the following recharging. Further, we show that the limited target cost work is together arched and infer the ideal arrangement. A mathematical model is introduced to outline the model and the affectability examination is additionally considered.

Keywords: Inventory, perishable items, exponential declining demand

Introduction

For the most part, in inferring the arrangement of monetary creation amount (EPQ) inventory model, we consider the interest rate and decay rate as steady amount. Yet, if there should be an occurrence of genuine issues, the interest rate and crumbling rate are not really steady however marginally upset from their unique fresh worth. The inspiration of this paper is to consider a more practical EPQ inventory model with limited creation rate and Weibull two boundaries decay. The spearheading research on inventory models is finished by Harris, In all inventory models an overall supposition that will be that items created have endlessly long lives. As a general rule, a large portion of the items decay over the long haul. There are numerous different items in reality that are dependent upon a critical pace of weakening. The effect of item decay ought not be ignored in the choice course of creation parcel size. Scientists are occupied with dissecting inventory models for deteriorating items like unpredictable fluids, prescriptions, electronic parts, style products, natural products, vegetables, and so forth A request level inventory model was created by Aggarwal (1978). Prior certain analysts like Ghare (1963) considered dramatically rotting inventory for a steady interest. Ideal creation anticipating a deteriorating thing was created by Hwang (1986), Lin(2005), and Raffat (1985). Planning of financial part size issue was contemplated by Maxwell (1964), Yang (2003) and Misra (1975). Time subordinate deteriorating thing with rescue esteem was created by Mishra (2008). Ideal arrangement for a deteriorating thing with limited recharging and with value subordinate interest rate was concentrated by Sabahno (2008). Shah (2008) has set up a model of time subordinate weakening with dramatic interest. An EPQ model for a passable postponement in installments of a thing with consistent decay was created by Sugapriya (2008). This part manages fostering an EPQ model for a solitary thing taking Weibull crumbling and variable holding cost. At the point when the items start falls apart value markdown is given to every unit of the thing.

ASSUMPTIONS AND NOTATIONS:

The assumptions taken in this model are given as follows:

(i) The demand rate for the product is known and finite.

(ii) Shortage is not allowed.

(iii) An infinite planning horizon is assumed.

(iv) Once a unit of the product is produced, it is available to meet the demand.

(v) Once the production is terminated the product starts deterioration and the price discount is considered.

(vi) There is no replacement or repair for a deteriorated item.

The notations that are employed here:

p: Production rate per unit time

d : Demand of the product per unit time.

A : Set up cost.

Where0 = $\alpha\beta t^{\beta-1}$ he Weibull two parameter deterioration rate (unit/unit time). Where , $0<\alpha<1$, $\beta >0$, α called scale parameter and p is called shape parameter.

Inventory carrying cost per unit time, where, h = a + bt, a and b are positive constants.

k: Production cost per unit.

r: Price discount per unit cost.

T: Cycle time.

T_1,: Production period.

T_2,: Time during which there is no production of the product i.e., T2=T-TX

I1(t) Inventory level for product during the production period, i.e. 0 <t

I2(t) Inventory level of the product during the period when there is no production, i.e. Tx<t

I(M): Maximum inventory level of the product.

TVC(T): Total cost/unit time.

MATHEMATICAL MODEL:

At time/=0i.e. at the underlying stage the inventory level is zero. Then, at that point, creation and supply start at the same time and the creation stops at t = Tx at which the greatest inventory 7(M) is reached. During this timeframe inventory developed at a rate p - d and there is no crumbling. After the clock, , the created units start disintegration and supply is proceeded at the rebate rate. As the interest stays consistent for the item the inventory level diminishes to nothing and afterward the creation run starts. Accordingly the inventory level of the item at time t over the period [o,r] can be addressed by the accompanying differential conditions

$dI_1(t)/dt = p - d \qquad 0 \leq t \leq T1$

………….1

And

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -d, T_1 \leq t \leq T$$

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Here $\theta=\alpha\beta t^{\beta-1}$ where $0 <\alpha <1$, $\beta>0$

α is called scale parameter and p is called shape parameter.

Here the boundary conditions are $I_1(0) = I_2T_2 = 0$

Using the boundary condition 7, (0) = 0, solution of equation (1) is

$I_1(t) = (p - d)t$, $0 \leq t \leq T_1$

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Integrating Factor of equation (2) is

e$\int \alpha\beta t\ \beta - 1\ = e^{\alpha t^{\beta}}$

Using the integrating factor solution of equation (2) is

$$I_2(t)e^{\alpha t^{\beta}} = -d\int e^{\alpha t^{\beta}}dt + c$$

Since 0 < a < 1, neglecting the terms involving second and higher powers of a in the expansion of exponential function we get,

$$I_2(t)e^{\alpha t^{\beta}} = -d\int \left(1 + \alpha t^{\beta}\right)dt + c$$
$$= -d\left(t + \frac{\alpha t^{\beta+1}}{\beta + 1}\right) + c$$

Now using the initial condition $I_2(T_2)$ =0 in the above we can find the required solution of equation (2) as

$I_2(t) = d\left(T_2 + \frac{\alpha T_2^{\beta+1}}{\beta+1} - t - \frac{\alpha t^{\beta+1}}{\beta+1}\right)e^{-\alpha t^{\beta}}$**……………4**

T1≤ t ≤T

The production cost per unit time is

PC = pk T_1/T

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The set up cost per unit time is

SC =A/T………….6

The Holding Cost is

$$HC = \frac{1}{T}\left[\int_0^{T_1} h(t)I_1(t)dt + \int_0^{T_2} h(t)I_2(t)dt\right]$$
$$= \frac{1}{T}\left[\int_0^{T_1} (a + bt)(p - d)tdt + d\int_0^{T_2} (a + bt)\left(T_2 + \frac{\alpha T_2^{\beta+1}}{\beta + 1} - t - \frac{\alpha t^{\beta+1}}{\beta + 1}\right)e^{-\alpha t^{\beta}}dt\right]$$

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Since 0 < a < 1, neglecting the terms involving second and higher powers of a in the expansion of exponential function we get,

$$HC = \frac{1}{T}\left[\int_0^{T_1}(a+bt)(p-d)tdt + d\int_0^{T_2}(a+bt)\left(T_2+\frac{\alpha T_2^{\beta+1}}{\beta+1}-t-\frac{\alpha t^{\beta+1}}{\beta+1}\right)\left(1-\alpha t^{\beta}\right)dt\right]$$

$$\Rightarrow HC = \left(\frac{p-d}{T}\right)_0^{T_1}(at+bt^2)dt$$

$$+\frac{d}{T}\int_0^{T_2}\left(a-a\alpha t^{\beta}+bt-b\alpha t^{\beta+1}\right)\left(T_2+\frac{\alpha T_2^{\beta+1}}{\beta+1}-t-\frac{\alpha t^{\beta+1}}{\beta+1}\right)dt$$

$$\Rightarrow HC = \frac{a(p-d)T_1^2}{2T}+\frac{b(p-d)T_1^3}{3T}+\frac{adT_2^2}{2T}+\frac{bdT_2^3}{6T}+\frac{da\alpha\beta T_2^{\beta+2}}{(\beta+1)(\beta+2)T}$$

$$+\frac{db\alpha\beta T_2^{\beta+3}}{2(\beta+3)(\beta+2)T}-\frac{da\alpha^2T_2^{2\beta+2}}{2(\beta+1)^2T}+\frac{db\alpha T_2^{2\beta+3}}{(\beta+1)T}\left(\frac{1}{2(\beta+3)}-\frac{\alpha}{\beta+2}\right)$$

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Let us express 7, and T2 in terms of F

We know /1(r,)=/,(0)

(p - d)T_1 =d $\left(T_2+\frac{\alpha T_2^{\beta+1}}{\beta+1}\right)$

To get a suitable result let us neglect the term containing a from the right hand side and then write

(p - d)T_1= dT_2

(p - d)T- T_2= dT_2

T2 = (p-d) T/p

T1 = dT /p

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Using these values of Tx and T2 in equation (62) we get

$$HC = \frac{a(p-d)d^2T}{2p^2} + \frac{b(p-d)d^3T^2}{3p^3} + \frac{da(p-d)^2T}{2p^2} + \frac{bd(p-d)^3T^2}{6p^3} + \frac{da\alpha\beta(p-d)^{\beta+2}T^{\beta+1}}{(\beta+1)(\beta+2)p^{\beta+2}} + \frac{db\alpha\beta(p-d)^{\beta+3}T^{\beta+2}}{2(\beta+3)(\beta+2)p^{\beta+3}} - \frac{da\alpha^2(p-d)^{2\beta+2}T^{2\beta+1}}{2(\beta+1)^2p^{2\beta+2}} + \frac{db\alpha(p-d)^{2\beta+3}T^{2\beta+2}}{(\beta+1)p^{2\beta+3}}\left(\frac{1}{2(\beta+3)} - \frac{\alpha}{\beta+2}\right)$$

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Deterioration cost:

The number of units that deteriorate in a cycle is the difference between the maximum inventory and the number of units used to meet the demand. Hence the deterioration cost per unit time is given as

$$DC = \frac{k}{T}\left[I_2(0) - \int_0^{T_2} d dt\right] = \frac{k}{T}\left[d\left(T_2 + \frac{\alpha T_2^{\beta+1}}{\beta+1}\right) - dT_2\right] = \frac{k}{T}\left(\frac{\alpha d T_2^{\beta+1}}{\beta+1}\right)$$

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Price discount:

Price discount is offered as a fraction of production cost for the units in the Period [o,T2]

PD = kr/T $\int_0^{T2} d dt = \frac{krdT_2}{T}$

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Therefore the average total cost per unit time is given by

$$TVC(T) = PC + SC + HC + DC + PD = \frac{pkT_1}{T} + \frac{A}{T} + \frac{a(p-d)T_1^2}{2T} + \frac{b(p-d)T_1^3}{3T} + \frac{adT_2^2}{2T} + \frac{bdT_2^3}{6T} + \frac{da\alpha\beta T_2^{\beta+2}}{(\beta+1)(\beta+2)T} + \frac{db\alpha\beta T_2^{\beta+3}}{2(\beta+3)(\beta+2)T} - \frac{da\alpha^2T_2^{2\beta+2}}{2(\beta+1)^2T} + \frac{db\alpha T_2^{2\beta+3}}{(\beta+1)T}\left(\frac{1}{2(\beta+3)} - \frac{\alpha}{\beta+2}\right) + \frac{k}{T}\left(\frac{\alpha d T_2^{\beta+1}}{\beta+1}\right) + \frac{krdT_2}{T}$$

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Using the values of T, and T2 from equations (10) and (9) in equation (13) we get

$$TVC(T) = kd + \frac{A}{T} + \frac{a(p-d)d^2T}{2p^2} + \frac{b(p-d)d^3T^2}{3p^3} + \frac{da(p-d)^2T}{2p^2} + \frac{bd(p-d)^3T^2}{6p^3} + \frac{da\alpha\beta(p-d)^{\beta+2}T^{\beta+1}}{(\beta+1)(\beta+2)p^{\beta+2}} + \frac{db\alpha\beta(p-d)^{\beta+3}T^{\beta+2}}{2(\beta+3)(\beta+2)p^{\beta+3}} - \frac{da\alpha^2(p-d)^{2\beta+2}T^{2\beta+1}}{2(\beta+1)^2p^{2\beta+2}} + \frac{db\alpha(p-d)^{2\beta+3}T^{2\beta+2}}{(\beta+1)p^{2\beta+3}}\left(\frac{1}{2(\beta+3)} - \frac{\alpha}{\beta+2}\right) + \frac{k\alpha d(p-d)^{\beta+1}T^{\beta}}{(\beta+1)p^{\beta+1}} + \frac{krd(p-d)}{p}$$

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To minimize the total cost, differentiating with respect to T and equating it to zero we get

$$\int \frac{d}{dt}(TVC(T) = 0$$

$$\Rightarrow -\frac{A}{T^2} + \frac{a(p-d)d^2}{2p^2} + \frac{b(p-d)d^3 2T}{3p^3} + \frac{da(p-d)^2}{2p^2} + \frac{bd(p-d)^3T}{3p^3} + \frac{da\alpha\beta(p-d)^{\beta+2}T^{\beta}}{(\beta+2)p^{\beta+2}} + \frac{db\alpha\beta(p-d)^{\beta+3}T^{\beta+1}}{2(\beta+3)p^{\beta+3}} - \frac{da\alpha^2(p-d)^{2\beta+2}(2\beta+1)T^{2\beta}}{2(\beta+1)^2p^{2\beta+2}} + \frac{2db\alpha(p-d)^{2\beta+3}T^{2\beta+1}}{p^{2\beta+3}} \times \left(\frac{1}{2(\beta+3)} - \frac{\alpha}{\beta+2}\right) + \frac{k\alpha d(p-d)^{\beta+1}\beta T^{\beta-1}}{(\beta+1)p^{\beta+1}} = 0$$

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Again $\int \frac{d2}{dt2}(TVC(T) = 0$

$$\frac{2A}{T^3} + \frac{2b(p-d)d^3}{3p^3} + \frac{bd(p-d)^3}{3p^3} + \frac{da\alpha\beta^2(p-d)^{\beta+2}T^{\beta-1}}{(\beta+2)p^{\beta+2}} + \frac{db\alpha\beta(p-d)^{\beta+3}(\beta+1)T^{\beta}}{2(\beta+3)p^{\beta+3}} - \frac{da\alpha^2(p-d)^{2\beta+2}(2\beta+1)\beta T^{2\beta-1}}{(\beta+1)^2p^{2\beta+2}} + \frac{2db\alpha(p-d)^{2\beta+3}(2\beta+1)T^{2\beta}}{p^{2\beta+3}}\left(\frac{1}{2(\beta+3)} - \frac{\alpha}{\beta+2}\right) + \frac{k\alpha d(p-d)^{\beta+1}\beta(\beta-1)T^{\beta-2}}{(\beta+1)p^{\beta+1}} > 0$$

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The values of Tfound from equation (15) minimize the total average variable cost if the second order derivative in equation (16) is positive for that.

NUMERICAL EXAMPLE:

Let A = Rs 4000 /set up, p =400 units/unit time, z/ — 100 unit/unit time, a = 6, 6 = 0.4, k = Rs80/unit, a = 0A,p = 0.8, r=0.04/unit. We used equation (16) to find that 2.01125 is the best value for T*. By plugging the value of r into equation 17, the value of the second order derivative, which is a positive number, was found to be 905.971. So, the total variable cost is kept as low as possible. Using the value of F* in equations (11) and (15), we get that HC* = 495.058 and TVC* = 12576.4..

SENSITIVITY ANALYSIS:

Now, you can do a sensitivity analysis of the model's best solution by changing one parameter at a time while leaving the others the same.

Table 1

Parameter	%change	Change in T	Change in HC	Change in TVC
A	-50	1.3826	336.003	11395.2
	-25	2.7214	395.765	12040.4
	0	2.5436	495.436	11345.3
	25	2.4980	560.875	12098.4
	50	2.4356	384.235	12300.3
p	-50	1.83461	**416.007**	11209.3
	-25	1.90146	**484.090**	12576.•3
	0	2.82152	**505.121**	12850.0
	25	*2.22492*	**479.001**	13032,4
	50	2.03171	**495.872**	12120.5

d	-50	2.5736	373.217	*1556.51*
	-25	2.19469	442.008	11180.3
	0	2.41115	495.058	12576.4
	25	1.90213	535.131	14803 .8
	50	1.88927	573.845	16894.7
k	-50	2.24855	554.714	10032.7
	-25	2.0155	495.059	12576.4
	0	1.83272	450.018	15083.5
	25	1.69225	414.533	17558.2
	50	2.58603		7483.2
α	-50	2.54938	630.299	11559,0
	-25	2.23552	552.761	11094.1
	0	1.01125	495.058	11576.4
	25	1.84011	449.6.12	13019.9
	50	1.70547	412.409	13433.1
β	-50	2.524938	612.91	12619.4
	-25	2.43552	540.893	12611.1
	0	1.01125	492.582	12576.4
	25	1.84011	464.092	12530.0
	50	1.70547	473.253	12480.1

From the above (Table) it tends to be deduced that the ideal process duration T* increments as the boundary An increments and it diminishes as the boundaries, d ,k, a, and p increments. Also the ideal holding cost HC* increments as the boundaries A ,p, d increments and it diminishes as the boundary k, a, and p increments. The ideal normal expense TVC* diminishes as the boundaries A,p ,d, k, and an abatements yet it increments as the parameter^ diminishes.

CONCLUSION:

In this paper, a Weibull disintegration rate inventory model with power design interest has been built. In this approach, flaws have been tolerated and, at times, accumulated. In contrast to the typical technique based on wide Weibull design, this kind of force design requirement necessitates an alternative strategy. We choose n > 1 when most of the interest happens at the beginning of the period, and n 1 when most of the interest happens at the end of the period. Similar to n=1 compared to steady and prompt interest, the number of participants is n=1.

Here an EPQ model has been created for a solitary machine, delivering single thing considering straight disintegration and variable holding cost concerning time. In this model totally decayed items have been disposed of and offered markdown for somewhat crumbled items which keeps up with the interest. The deficiencies are not permitted in this model. The ideal creation process duration has been inferred. This review assists with limiting the complete expense for direct crumbling. Here in this model it is shrewd to diminish the worth ofthe boundary p or d or a to limit the absolute expense for a thing. Further review should be possible taking different requests or weakening for this model.

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